

## On the Newton's method for the minimal non-negative solution of the non-symmetric coupled algebraic Riccati equation

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### Abstract

In this paper, we consider the Newton's method for computing the minimal non-negative solution to the following non-symmetric coupled Riccati equations (NCAREs):

$$R_i(X_1, \dots, X_i, \dots, X_s) = X_i C_i X_i - X_i D_i - A_i X_i + B_i + \sum_{j \neq i} e_{ij} X_j = 0, \quad i = 1, 2, \dots, s,$$

where  $X_i \in \mathbb{R}^{m \times n}$ ,  $B_i \in \mathbb{R}^{m \times n}$ ,  $C_i \in \mathbb{R}^{n \times m}$ , and  $D_i \in \mathbb{R}^{n \times n}$ ,  $i = 1, 2, \dots, s$  are matrix coefficients, and  $e_{ij}$ ,  $i = 1, 2, \dots, s$ ,  $j = 1, 2, \dots, s$  are non-negative constants.

Based on the theory of the non-symmetric algebraic Riccati equation and the coupled Riccati equations the NCAREs have been investigated in [1]. In [1] the authors present two iterative methods, which are considered in [2] – a fixed point iteration and a method called “Newton iteration”, two numerical iteration methods for single non-symmetric algebraic Riccati equation – inexact Newton method (INewton) [3] and alternate linear implicit method (ALI) [4] for NCAREs are proposed, the convergence theory of the two iteration methods are given and finally some numerical examples are offered.

Motivated by the investigations in [1] and presented numerical results for convergence behavior of considered methods, we study the Newton's method and show that the presented “Newton iteration in [2] is an alternatively method. In addition, we consider some numerical examples to illustrate the theoretical results.

Key words: non-symmetric coupled Riccati equations, minimal non-negative solution, iterative methods.

### References

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